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ANALYTIC SOLUTION FOR INTERLAMINAR STRESSES IN A MULTILAMINATED CYLINDRICAL SHELL UNDER THERMAL AND MECHANICAL LOADS

X. WANG
Department of Applied Mechanics, East China Institute of Technology, Nanjing, 210014, China

and

S. J. LI
Department of Engineering Mechanics, Shanghai Jiao Tong University, Shang Hai, China

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Abstract—An analytical method is presented for the problem of interlaminar stresses in a cross-ply multilaminated cylindrical shell subject to thermal loading and radial pressure with various end boundary conditions. The interlaminar stresses are determined exactly by using the method of separation of variables in three-dimensional theory of anisotropic elasticity. The results obtained can not only be used as a reference for approximate theories, but are also valuable for application in the design of composite structures.

INTRODUCTION

The problem of interlaminar stresses in laminated plates and shell structures continues to be a popular topic for intensive research. Recently, a number of analytical solutions, mainly concerned with free edges of a laminated plate, have been presented. A good example of one of these solutions was presented by Kassapoglou and Lagace (1986). This method of solution, based on the overall force and moment equilibrium and the principle of minimum complementary energy, calculates the three-dimensional stress state in laminated plates. However, so far, we have not found this method to be suitable for determining the three-dimensional stress state in a finite long laminated shell. A higher order shell theory is often used to determine the interlaminar stresses in laminated shells subjected to mechanical or thermal loadings, such as the works of Zukas and Vinson (1971), Padovan and Lestingi (1972), Zukas (1974) and Waltz and Vinson (1976). An analytical solution for the interlaminar stresses in a fiber-reinforced double-layer cylindrical shell under internal pressure was obtained by Li *et al.* (1985). To determine interlaminar stresses in laminated shells is more complicated than in laminated plates. It is valuable to present an exact analytical method to solve the interlaminar stresses in a laminated cylindrical shell with s laminae.

In this paper, a method is developed to determine the interlaminar stresses in an arbitrarily thick, cross-ply multilaminated cylindrical shell subjected to axisymmetrically distributed mechanical and thermal loadings with simply supported, clamped end boundary conditions or given axial displacement at ends. By utilizing the theory of three-dimensional anisotropic elasticity and the method of separation of variables, equilibrium equations with unknown displacements for each cylindrical lamina are derived and solved. Then, making the displacement and stress expressions satisfy the boundary conditions and the continuity conditions at the interfaces of the plies, the interlaminar stresses are determined exactly.

The results obtained can not only be used as a basis for the assessment of the results obtained from various approximate theories, but can also be used in the design of composite structures.

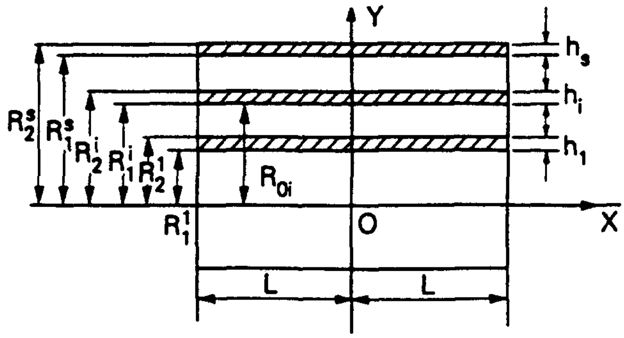

Fig. 1. The geometry and coordinate system.

FUNDAMENTAL EQUATIONS AND SOLUTIONS

The geometry and coordinate system of laminated shells is shown in Fig. 1. x, r and θ represent the axial, radial and tangential variables respectively. The ordinal number of laminated shells is denoted by a subscript or a superscript i, ($i = 1, \ldots, 2, s$). h_i, R_1^i, R_2^i and R_{0i} represent the thickness, the internal radii, the external radii and the characteristic length of the ith lamina respectively.

Hooke's Law for an orthotropic cylindrical shell under thermal loading in an axisymmetrical problem may be written as:

$$\begin{bmatrix} \sigma_x^i \\ \sigma_r^i \\ \sigma_\theta^i \\ \tau_{xr}^i \end{bmatrix} = C_{22}^i \begin{bmatrix} C_5^i & C_2^i & C_4^i & 0 \\ C_2^i & 1 & C_1^i & 0 \\ C_4^i & C_1^i & C_3^i & 0 \\ 0 & 0 & 0 & C_6^i \end{bmatrix} \begin{bmatrix} \varepsilon_x^i - \alpha_x^i T^i \\ \varepsilon_r^i - \alpha_r^i T^i \\ \varepsilon_\theta^i - \alpha_\theta^i T^i \\ \gamma_{xr}^i \end{bmatrix} \tag{1}$$

where

$$C_1^i = \frac{C_{23}^i}{C_{22}^i}, \quad C_2^i = \frac{C_{12}^i}{C_{22}^i}, \quad C_3^i = \frac{C_{33}^i}{C_{22}^i}, \quad C_4^i = \frac{C_{13}^i}{C_{22}^i},$$
$$C_5^i = \frac{C_{11}^i}{C_{22}^i}, \quad C_6^i = \frac{G^i}{C_{22}^i}. \tag{2a-f}$$

The principal directions (1, 2, 3) of the material in [0]-ply are consistent with the coordinate axes (x, r, θ). Thus the stiffness components are shown as:

$$C_{jk}^i = C_{jk}, \quad (j, k = 1, 2, 3), \quad G^i = G_{12}.$$

The thermal expansion coefficients are specified as:

$$\alpha_r^i = \alpha_\theta^i = \alpha_{22} = \alpha_{33} = \alpha_T, \quad \alpha_x^i = \alpha_{11} = \alpha_L.$$

For [90]-ply, the stiffness components are written as:

$$C_{11}^i = C_{33}, \quad C_{12}^i = C_{23}, \quad C_{13}^i = C_{13}$$
$$C_{22}^i = C_{22}, \quad C_{33}^i = C_{11}, \quad C_{23}^i = C_{12}, \quad G^i = G_{23}.$$

The thermal expansion coefficients are shown as:

$$\alpha_x^i = \alpha_r^i = \alpha_{33} = \alpha_{22} = \alpha_T, \quad \alpha_\theta^i = \alpha_{11} = \alpha_L.$$

$T^i = \Delta t$ represents the temperature variable of the ith lamina.

By making use of the geometric relations and equilibrium equation of the axisymmetrical problem, the fundamental differential equations for each lamina with unknown displacement can be written as:

$$\frac{\partial^2 \bar{U}^i}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{U}^i}{\partial r} - C_3^i \frac{\bar{U}^i}{r^2} + C_6^i \frac{\partial^2 \bar{U}^i}{\partial x^2} + (C_2^i + C_6^i)\frac{\partial^2 \bar{W}^i}{\partial r \partial x} + (C_2^i - C_4^i)\frac{1}{r}\frac{\partial \bar{W}^i}{\partial x} + \alpha_c^i T^i \frac{1}{r} = 0 \tag{3a}$$

$$\frac{\partial^2 \bar{W}^i}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{W}^i}{\partial r} + \frac{C_5^i}{C_6^i}\frac{\partial^2 \bar{W}^i}{\partial x^2} + \left(1 + \frac{C_2^i}{C_6^i}\right)\frac{\partial^2 \bar{U}^i}{\partial r \partial x} + \left(1 + \frac{C_4^i}{C_6^i}\right)\frac{1}{r}\frac{\partial \bar{U}^i}{\partial x} = 0 \tag{3b}$$

where

$$\alpha_c^i = (C_1^i - 1)\alpha_r^i + (C_3^i - C_1^i)\alpha_\theta^i + (C_4^i - C_2^i)\alpha_x^i. \tag{3c}$$

$\bar{U}^i$ and $\bar{W}^i$ represent the radial displacement and the axial displacements respectively.

If we define η_i as the radial local coordinate with respect to the middle surface of the ith lamina and take each lamina as a thin shell, then we will have $\eta_i/R_{0i} \ll 1$ and the following formulae:

$$\eta_i = r - R_{0i} \tag{4a}$$

$$\frac{1}{r} = \frac{1}{R_{0i}}\left(1 - \frac{\eta_i}{R_{0i}}\right) \tag{4b}$$

$$\frac{1}{r^2} = \frac{1}{(R_{0i})^2}\left(1 - 2\frac{\eta_i}{R_{0i}}\right). \tag{4c}$$

By substituting (4) into (3), omitting the terms of the same order as η_i/R_{0i} or higher and taking $\xi_i = \eta_i/R_{0i}$, we obtain

$$\left[\frac{\partial^2}{\partial \xi_i^2} + \frac{\partial}{\partial \xi_i} - C_3^i + C_6^i R_{0i}^2 \frac{\partial^2}{\partial x^2}\right]\bar{U}^i + \frac{\partial}{\partial x}\left[(C_2^i + C_6^i)R_{0i}\frac{\partial}{\partial \xi_i} + R_{0i}(C_2^i - C_4^i)\right]\bar{W}^i + R_{0i}\alpha_C^i T^i = 0 \tag{5a}$$

$$\left[\frac{\partial^2}{\partial \xi_i^2} + \frac{\partial}{\partial \xi_i} + R_{0i}^2 \frac{C_5^i}{C_6^i}\frac{\partial^2}{\partial x^2}\right]\bar{W}^i + R_{0i}\frac{\partial}{\partial x}\left[\left(1 + \frac{C_2^i}{C_6^i}\right)\frac{\partial}{\partial \xi_i} + \left(1 + \frac{C_4^i}{C_6^i}\right)\right]\bar{U}^i = 0. \tag{5b}$$

A inhomogeneous solution of eqn (5) may be takes as:

$$U_0^i = R_{0i}\alpha_C^i T^i \frac{1}{C_3^i} + R_{0i}(C_2^i - C_4^i)W_C^i \frac{1}{C_3^i} \tag{6a}$$

$$W_0^i = W_C^i \cdot x \tag{6b}$$

where W_C^i are unknown constants. The homogeneous solution of eqn (5) is prescribed as:

$$U^i = \sum_n f_n^i(\xi_i)\cos\alpha_n x \tag{7a}$$

$$W^i = \sum_n g_n^i(\xi_i)\sin\alpha_n x. \tag{7b}$$

Substitution of (7) into (5) yields a corresponding homogeneous form of eqn (5)

$$(D^2+D+a_{1n}^i)f_n^i(\xi_i)+(a_{2n}^i D+a_{3n}^i)g_n^i(\xi_i) = 0 \tag{8a}$$

$$(D^2+D+a_{4n}^i)g_n^i(\xi_i)-(a_{5n}^i D+a_{6n}^i)f_n^i(\xi_i) = 0 \tag{8b}$$

where

$$D = \frac{\mathrm{d}}{\mathrm{d}\xi_i}, \quad D^2 = \frac{\mathrm{d}^2}{\mathrm{d}\xi_i^2}$$

$$a_{1n}^i = -(C_3^i+C_6^i R_{0i}^2\alpha_n^2), \quad a_{2n}^i = (C_2^i+C_6^i)R_{0i}\alpha_n$$

$$a_{3n}^i = (C_2^i-C_4^i)R_{0i}\alpha_n, \quad a_{4n}^i = -R_{0i}^2\alpha_n^2\frac{C_5^i}{C_6^i}$$

$$a_{5n}^i = \left(1+\frac{C_2^i}{C_6^i}\right)R_{0i}\alpha_n, \quad a_{6n}^i = \left(1+\frac{C_4^i}{C_6^i}\right)R_{0i}\alpha_n. \tag{8c–j}$$

By defining the eigenfunction of $f_n^i(\xi_i)$ and $g_n^i(\xi_i)$ as $e^{\lambda_n^i\xi_i}$, the eigenequation of eqn (8) can be written as:

$$(\lambda_n^i)^4+2(\lambda_n^i)^3+b_{1n}^i(\lambda_n^i)^2+(b_{1n}^i-1)\lambda_n^i+b_{2n}^i = 0 \tag{9a}$$

or

$$[(\lambda_n^i)^2+\lambda_n^i+b_n^i]\cdot[(\lambda_n^i)^2+\lambda_n^i+d_n^i] = 0 \tag{9b}$$

where

$$b_{1n}^i = 1-C_3^i-R_{0i}^2\alpha_n^2(C_5^i-C_2^i-2C_2^iC_6^i)/C_6^i \tag{9c}$$

$$b_{2n}^i = C_5^iR_{0i}^4\alpha_n^4+R_{0i}^2\alpha_n^2(C_3^iC_5^i+C_2^iC_6^i+C_2^iC_4^i-C_4^iC_6^i-C_4^iC_4^i)/C_6^i \tag{9d}$$

$$b_n = \tfrac{1}{2}(b_{1n}^i-1)+\tfrac{1}{2}\sqrt{(b_{1n}^i-1)^2-4b_{2n}^i} \tag{9e}$$

$$d_n = \tfrac{1}{2}(b_{1n}^i-1)-\tfrac{1}{2}\sqrt{(b_{1n}^i-1)^2-4b_{2n}^i}. \tag{9f}$$

The exact solution of eqn (8) can be obtained by solving eqn (9). Thereby the expressions for displacements and stresses are gained.

In formula (6), T^i and W_C^i can also be rewritten as:

$$T^i = \sum_n T^i\delta_n\cos\alpha_n x, \quad W_C^i = \sum_n W_C^i\delta_n\cos\alpha_n x \tag{10a–b}$$

where

$$\delta_n = \frac{\int_{-L}^{L} \cos \alpha_n x \, dx}{\int_{-L}^{L} \cos^2 \alpha_n x \, dx}. \tag{10c}$$

In the case of $(b^i_{1n}-1)^2 > 4b^i_{2n}$, eqn (9) has four different real roots:

$$\lambda^i_{(1,2)n} = -\tfrac{1}{2} \pm \tfrac{1}{2}\sqrt{1-4b^i_n}, \quad \lambda^i_{(3,4)n} = -\tfrac{1}{2} \pm \tfrac{1}{2}\sqrt{1-4d^i_n}. \tag{11a–d}$$

From eqns (6), (7) and (10), the expressions for displacements and stresses are found:

$$\bar{U}^i = U^i_0 + U^i = \sum_n \left[\sum_{l=1}^{4} e^{\lambda^i_{ln}\xi_i} A^i_{ln} + \frac{R_{0i}\alpha^i_c T^i \delta_n}{C^i_3} + \frac{R_{0i}}{C^i_3}(C^i_2 - C^i_4) W^i_C \delta_n \right] \cos \alpha_n x$$

$$\bar{W}^i = W^i_0 + W^i = \sum_n \left[\sum_{l=1}^{4} - cm^i_{ln} e^{\lambda^i_{ln}\xi_i} A^i_{ln} \right] \sin \alpha_n x + W^i_C x$$

$$\sigma^i_r = C^i_{22} \sum_n \left[\sum_{l=1}^{4} \left(\frac{\lambda^i_{ln}}{R_{0i}} + \frac{C^i_1}{r} - C^i_2 \alpha_n cm^i_{ln} \right) e^{\lambda^i_{ln}\xi_i} A^i_{ln} + \frac{R_{0i} C^i_1}{r C^i_3} \right.$$
$$\left. \cdot (C^i_2 - C^i_4) W^i_C \delta_n + C^i_2 W^i_C \delta_n + CT^i_r \right] \cos \alpha_n x$$

$$\sigma^i_\theta = C^i_{22} \sum_n \left[\sum_{l=1}^{4} \left(\frac{\lambda^i_{ln}}{R_{0i}} C^i_1 + \frac{C^i_3}{r} - C^i_4 \alpha_n cm^i_{ln} \right) e^{\lambda^i_{ln}\xi_i} A^i_{ln} + \frac{R_{0i}}{r} \right.$$
$$\left. \cdot (C^i_2 - C^i_4) W^i_C \delta_n + CT^i_\theta \right] \cos \alpha_n x + C^i_{22} C^i_4 W^i_C$$

$$\sigma^i_x = C^i_{22} \sum_n \left[\sum_{l=1}^{4} \left(\frac{\lambda^i_{ln}}{R_{0i}} C^i_2 + \frac{C^i_4}{r} - C^i_5 \alpha_n cm^i_{ln} \right) e^{\lambda^i_{ln}\xi_i} A^i_{ln} + \frac{R_{0i} C^i_4}{r C^i_3} \right.$$
$$\left. \cdot (C^i_2 - C^i_4) W^i_C \delta_n + CT^i_x \right] \cos \alpha_n x + C^i_{22} C^i_5 W^i_C$$

$$\tau^i_{xr} = G^i \sum_n \left[\sum_{l=1}^{4} - \left(\frac{\lambda^i_{ln}}{R_{0i}} cm^i_{ln} + \alpha_n \right) e^{\lambda^i_{ln}\xi_i} A^i_{ln} - \frac{\alpha_n R_{0i}}{C^i_3} (C^i_2 - C^i_4) W^i_C \delta_n \right.$$
$$\left. - \frac{1}{C^i_3} \alpha_n R_{0i} \alpha^i_c T^i \delta_n \right] \sin \alpha_n x \tag{12a–f}$$

where

$$cm^i_{ln} = \frac{(\lambda^i_{ln}\lambda^i_{ln} + \lambda^i_{ln} + a^i_{ln})}{(a^i_{2n}\lambda^i_{ln} + a^i_{3n})}, \tag{12g}$$

CT^i_r, CT^i_θ, CT^i_x represent respectively the known constants.

In the case of $(b^i_{1n}-1)^2 < 4b^i_{2n}$, eqn (9) has two pairs of conjugate complex roots:

$$\lambda^i_{(1,2)_n} = I^i_n(b^i_{1n}, b^i_{2n}) \pm i^* \cdot g^i_n(b^i_{1n}, b^i_{2n})$$
$$\lambda^i_{(3,4)_n} = J^i_n(b^i_{1n}, b^i_{2n}) \pm i^* \cdot g^i_n(b^i_{1n}, b^i_{2n}) \tag{13a–d}$$

where

$$i^* = \sqrt{-1}.$$

The other expressions for displacements and stresses can also be obtained. For brevity, they are omitted herein.

For each individual laminate, according to two kinds of different eigenroots (11) and (13), a set of the displacements and stresses can be found. It involves just $4n+1$ unknown constants. For a laminated shell with s laminae, the total unknowns are $(4n+1)s$ $(A^i_{1n}, A^i_{2n}, A^i_{3n}, A^i_{4n}, w^i_{C'}, i = 1, 2, 3, \ldots, s)$, which can be determined from the given boundary conditions and the interface continuity conditions.

BOUNDARY AND CONTINUITY CONDITIONS

(I) On internal and external surfaces:

$$\left(r_1 = \frac{h_1}{2R_{01}},\quad r_s = \frac{h_s}{2R_{0s}}\right):$$

$$\sigma^1_r = -p_1(x),\quad \sigma^s_r = -p_2(x),\quad \tau^1_{xr} = 0,\quad \tau^s_{xr} = 0. \tag{14}$$

(II) On both ends ($x = \pm L$) the boundary conditions are

$$\bar{U}^i = 0,\quad \sigma^i_x = 0,\quad (i = 1, 2, \ldots, s)\quad \text{for simply supported} \tag{15}$$

or

$$\bar{U}^i = 0,\quad \bar{W}^i = W_0,\quad (i = 1, 2, \ldots, s) \tag{16}$$

for a given axial displacement at ends.

(III) The continuity conditions

$$\left(r_i = \frac{h_i}{2R_{0i}}, r_{i+1} = \frac{h_{i+1}}{2R_{0i+1}}\right):$$

$$\sigma^i_r = \sigma^{i+1}_r,\quad \tau^i_{xr} = \tau^{i+1}_{xr}$$
$$\bar{U}^i = \bar{U}^{i+1},\quad \bar{W}^i = \bar{W}^{i+1}. \tag{17}$$

From (15) or (16), we have

$$\cos[\alpha_n(\pm L)] = 0. \tag{18a}$$

Thus

$$\alpha_n = \frac{(2n-1)\pi}{2L}\quad n = 1, 2, \ldots, \infty. \tag{18b}$$

Thereby in eqn (10), δ_n is rewritten as:

$$\delta_n = \frac{-2(-1)^n}{\alpha_n L}. \tag{19}$$

From (15) and (12e), we obtain

$$W_C^i = 0. \tag{20}$$

When λ_n^i are four different real roots, from eqns (16) and (12b), we have:

$$W_C^i = \frac{W_0}{L} + \frac{1}{L}\sum_n \left[(-1)^{n-1} \sum_{l=1}^{4} cm_{ln}^i \, e^{\lambda_{ln}^i \xi_i} A_{ln}^i \right] \tag{21}$$

or

$$W_C^i L - W_0 = \sum_n \left[(-1)^{n-1} \sum_{l=1}^{4} cm_{ln}^i \, e^{\lambda_{ln}^i \xi_i} A_{ln}^i \right]. \tag{22}$$

Let

$$W_C^i L - W_0 = \frac{4}{\pi}\sum_n (-1)^{n-1} \frac{1}{(2n-1)} (W_C^i L - W_0). \tag{23}$$

From eqns (22) and (23), W_C^i can be rewritten as:

$$W_C^i = \frac{\pi(2n-1)}{4L} \sum_{l=1}^{4} cm_{ln}^i \, e^{\lambda_{ln}^i \xi_i} A_{ln}^i + \frac{W_0}{L}. \tag{24}$$

Obviously, eqns (21) and (24) are equal to W_C^i. When λ_n^i are two pairs of conjugate complex roots, we may also obtain two expressions for W_C^i. For brevity, it is omitted herein.

Making use of eqns (21) or (24) and (14)–(18), we have $(4n+1)s$ equations, which determine the $(4n+1)s$ unknown constants completely. Thus the displacement and stress fields are obtained exactly.

EXAMPLES

As an example, we calculate a four-layer laminated shell with Boron/Epoxy fiber-reinforced composite material [90/m/0/m] and [0/m/90/m], where m is taken as a bonding layer of Epoxy. For ease of comparison with other references, the material properties are specified as:

$$E_{11} = 228\,\text{GPa}, \quad E_{22} = E_{33} = 12.95\,\text{GPa}, \quad G_{12} = 4.52\,\text{GPa}$$

$$G_{23} = 2.45\,\text{GPa}, \quad \nu_{23} = \nu_{32} = 0.25, \quad \nu_{21} = \nu_{31} = 0.0145$$

$$\nu_{12} = \nu_{13} = 0.256, \quad \alpha_{22} = \alpha_{33} = \alpha_T = 10 \cdot 10^{-6}\,{}^\circ\text{F}^{-1}$$

$$\alpha_{11} = \alpha_L = 10^{-6}\,{}^\circ\text{F}^{-1}.$$

Epoxy resin is taken to be isotropic and the material properties are:

$$E = 3.52\,\text{GPa}, \quad G = 1.3\,\text{GPa}, \quad \nu = 0.35, \quad \alpha = 5 \cdot 20^{-6}\,{}^\circ\text{F}^{-1}.$$

The kinds of loading are specified respectively as follows:

1. Temperature variable in each lamina: $T^i = \Delta t$.
2. Internal pressure: $p = p_1$.
3. Axial displacement at end: $W_0 = -0.1$ cm.

The dimensions of laminated shell are taken as

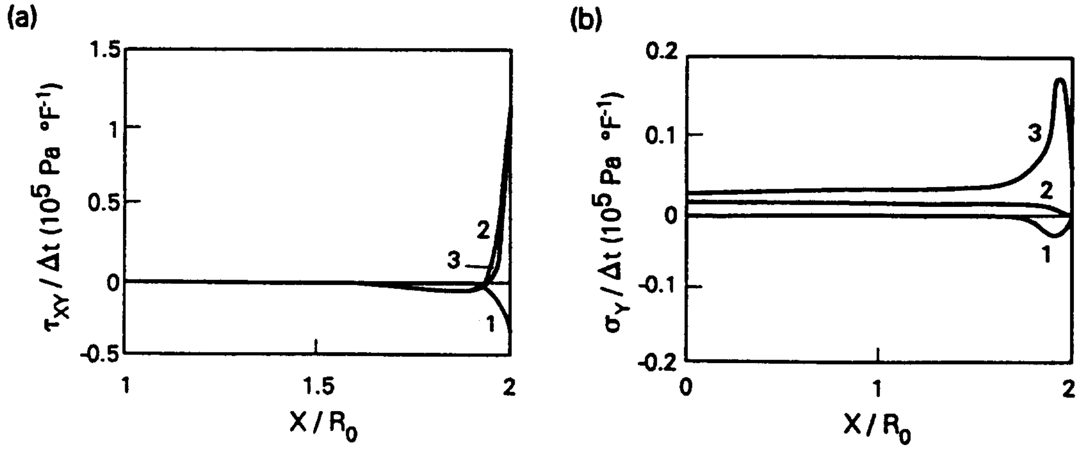

Fig. 2. [90/m/0/m] Thermal load, simply supported ends.

$$h_1 = h_2 = h_3 = h_4, \quad h = h_1 + h_2, \quad R_0 = \frac{1}{2}(R_1^1 + R_2^4), \quad \frac{h}{R_0} = \frac{1}{50}, \quad \frac{L}{R_0} = 2.$$

τ_{xr}^{l} in the figures represent the maximum shear stress at the ends of the cylindrical shell.

DISCUSSIONS AND CONCLUSIONS

(1) That the interlaminar stresses due to thermal load are as strong as that due to mechanical load is shown in Fig. 2. The results show that the edge effects appear obviously and the maximum is at the ends. The peak of the interlaminar shear stress between the second and third layers is the greatest in the radial direction. It is noted that the interlaminar normal stresses between the second and third layers, and the third and fourth layers are stretching and it is a cause of delamination in the stress concentration region.

(2) Laminated patterns have also greatly influenced the interlaminar stress distributions. It is seen from Figs 2 and 3 that the interlaminar stresses in [90/m/0/m] are greater than that in [0/m/90/m]. Therefore the loading capacity of laminated structures can be increased greatly by the correct choice of ply patterns.

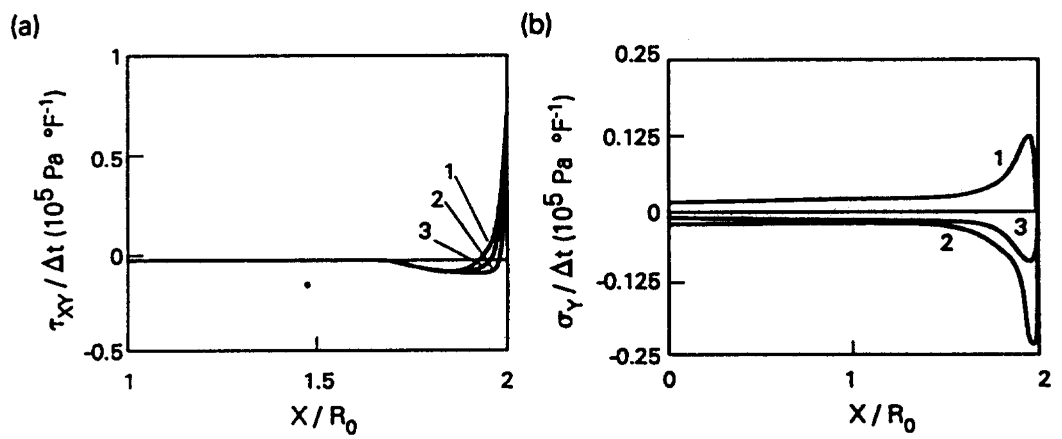

Fig. 3. [0/m/90/m] Thermal load, simply supported ends.

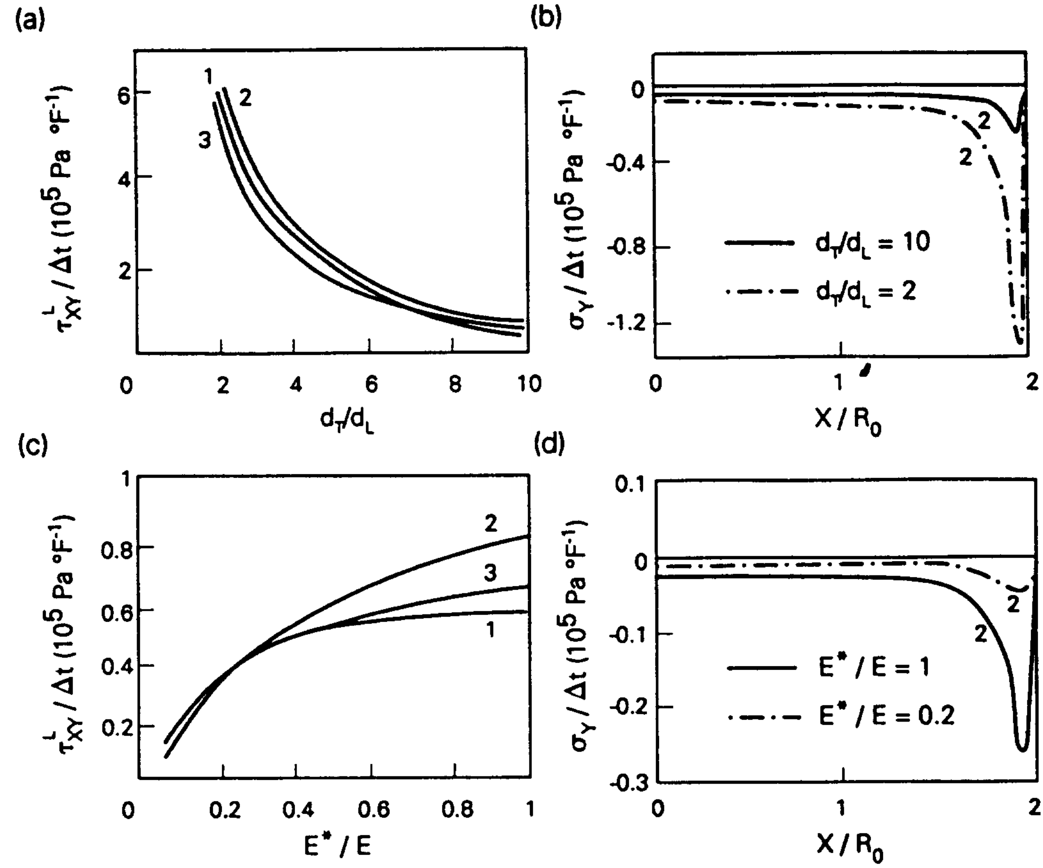

Fig. 4. [0/m/90/m] Thermal load, simply supported ends, the influence of the longitudinal thermal expansion coefficient and matrix elastic modulus.

(3) It is seen from Fig. 4a–b that the less are the longitudinal thermal expansion coefficients, the less are the interlaminar stresses when the transverse thermal expansion coefficients of the fiber, $\alpha_{22} = \alpha_{33} = \alpha_r$, are given.

(4) It is shown in Figs 4c–d and 5 that the interlaminar stress peak can be decreased greatly when the matrix is made of a material with low elastic modulus E^*. Thereby we

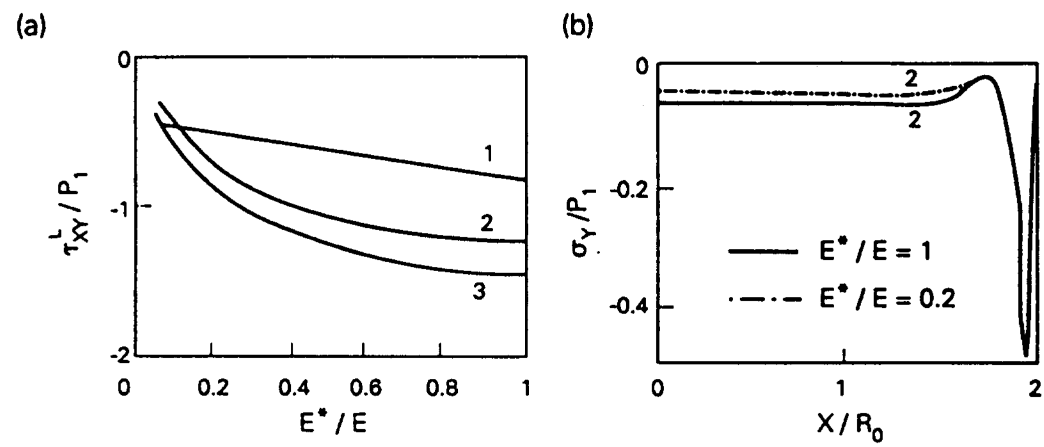

Fig. 5. [0/m/90/m] Internal pressure, simply supported ends.

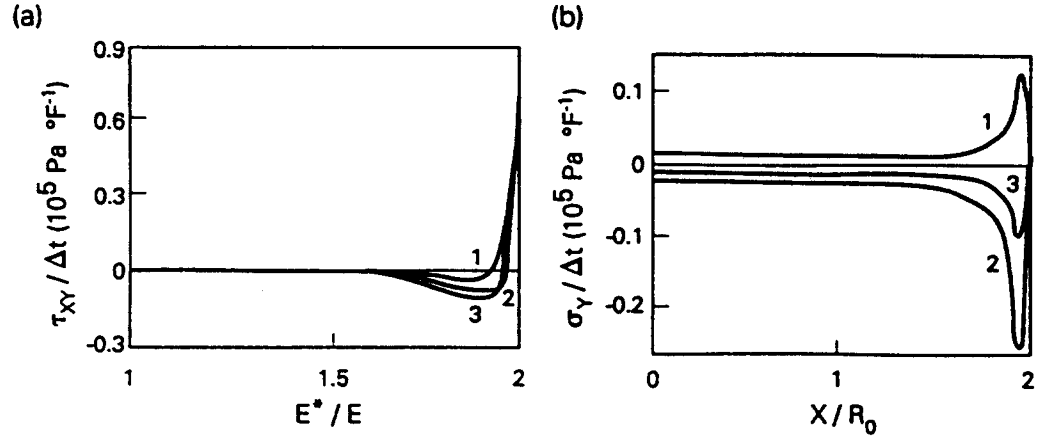

Fig. 6. [0/m/90/m] Thermal load, clamped ends.

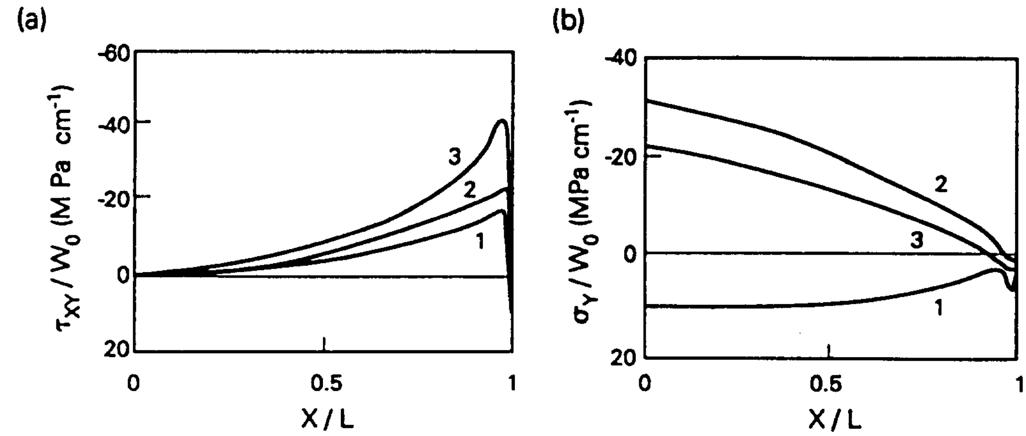

Fig. 7. [0/m/90/m] Axial compression.

can reach the conclusion that the strength of the laminated structure with lower elastic modulus and high strength matrix can be increased greatly.

(5) From Fig. 6, we can see that the interlaminar stress peak and fluctuating region at the clamped ends is greater than that at the simply supported ends.

(6) It is seen from Fig. 7 that the interlaminar stresses of the laminated shell with the compression of given axial displacement fluctuated all along the shell. The shear stress peak at the end is the greatest. The normal stresses appear as stretching regions between the second and third layers and the third and fourth layers.

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